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A NEW DAMAGE MECHANICS BASED APPROACH TO FATIGUE LIFE PREDICTION AND ITS ENGINEERING APPLICATION**

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ABSTRACT An approach based on continuum damage mechanics to fatigue life prediction for structures is proposed. A new fatigue damage evolution equation is developed, in which the parameters are obtained in a simple way with reference to the experimental results of fatigue tests on standard specimens. With the utilization of APDL language on the ANSYS platform, a finite element implementation is presented to perform coupling operation on damage evolution of material and stress redistribution. The fatigue lives of some notched specimens and a Pitch-change-link are predicted by using the above approach. The calculated results are validated with experimental data.

KEY WORDS fatigue damage model, damage mechanics, fatigue life, finite element method

I. INTRODUCTION

Since the fatigue failure as a common form of failure in a metallic structure is a well-known technical problem, it is of great importance to predict it for real engineering structures. However, this is no easy job because a great number of factors can affect the fatigue life, of which two crucial ones are always considered by researchers. One is the fatigue property of material, and the other the stress distribution in the structure. The fatigue property of material is described by stress versus number of cycles of failure (S-N) curves obtained by fatigue experiments on standard specimens. The distribution of stress field can be calculated by the finite element method (FEM). As most engineering components are subjected to multiaxial fatigue loads and the history of a loading cycle is very complicated , the stress distribution is different from that of standard specimens. Despite considerable standard specimen experiments , the relationship between the fatigue behavior of standard specimen and engineering structure is still hard to determine.

Numerous methods have been adopted to predict the fatigue safety of the components. Stress equivalent[1] and stress invariant[2] approaches make use of the S-N curves with information about the stress field to predict the fatigue life. The critical plane approach, proposed by Findley[3], Fatemi and Socie[4] and McDiarmid[5], has found wide application . Szolwinski and Farrisu[6] have adopted the critical method in researches on fretting fatigue problems. This method searches for the maximum value of a fatigue damage parameter (e.g. SWT, the Smith-Watson-Topper parameter) over a number

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of different planes and predicts the fatigue life for the worst damaged plane. The integral approach[7] based on the critical plane approach integrates a fatigue parameter over a characteristic depth or volume to make a rational prediction of fatigue life. The critical plane approach is attractive from a mechanical standpoint, as criteria developed within this framework provide not only a fatigue strength estimate of the component, but also the location and direction expected of early crack initiation. However, the method is based on physical observation that fatigue cracks initiate within a material on certain planes and employs some empirical equations. The evolution of fatigue damage is not clearly pointed out in these methods.

A continuum damage mechanics approach has been proposed[8–11], which treats the mechanical behavior of a deteriorated medium on the macroscopic scale without requiring determination of a critical plane and any loading cycle counting algorithm to calculate the fatigue life, since it operates by simply evaluating the progressive damage effects accumulated within the material during a variable loading history. The key point of the approach is build a damage evolution equation to describe the process of fatigue damage and identify the parameters in the equation. Recently, Marmi[12] and Zhang[13] have investigated the Lemaitre and Chaboche damage model to predict life not only for plain fatigue but also fretting fatigue. However the model is complicated and the identification of the five parameters needs a number of fatigue tests.

This research presents a new fatigue model to deal with the high cycle fatigue (HCF) problem. A new damage evolution equation is proposed on the basis of thermodynamics. The parameters in the damage evolution equation are conveniently obtained with reference to the fatigue experimental data on the smooth specimens. Then the damage mechanics-finite element method is built for the ANSYS platform to calculate the fatigue life of notched specimens and the Pitch-Change-Link[14], in which the coupling effect between the fatigue damage of material and stress distribution in structure is taken into account. The predicted results of fatigue lives of notched specimens and the Pitch-Change-Link are validated with experimental data, which indicates that the combination of damage mechanics-finite element method and fatigue tests on standard smooth specimens can provide a rational fatigue life prediction for engineering structures.

II. FATIGUE DAMAGE MODEL

2.1. Damage Variable

Damage in its mechanical sense is the creation and growth of micro-cracks or micro-voids, which are discontinuities in a medium considered continuous on a large scale[15]. In engineering, the mechanics of continuous media can be described by a Representative Volume Element (RVE) in which all properties are represented by homogenized variables, as shown in Fig.1[16]. By assuming the isotropic damage, the scalar damage variable D is defined as

$$D = \frac{S_D}{S} = \frac{S - S_R}{S} \tag{1}$$

where S, S_D, S_R are the total area of the section in the RVE, the total area of micro-cracks or micro-voids and the effective area of resistance, respectively. $S_R = S - S_D = (1 - D)S$. The effective stress $\tilde{\sigma}$ is introduced to describe stress over the section, which effectively resists the forces

$$\tilde{\sigma} = \frac{P}{S_R} = \frac{P}{(1-D)S} = \frac{\sigma}{1-D} \tag{2}$$

where σ is the stress for the undamaged material. Damage in the material is accumulated with increased loading cycles. The initiation of macro-cracks takes place once the damage reaches a critical value. The critical value of accumulated damage is defined to be 1 in the present study. At that moment, the effective area of resistance, S_R, decreases to zero and the effective stress $\tilde{\sigma}$ tends to infinity, which signals the initiation of macro-cracks.

The damage variable is coupled into the constitutive model by using the effective stress instead of the stress, which is in conformity with the strain equivalent principle[10, 17–20].

$$\varepsilon_{ij} = \frac{1+\nu}{E}\left(\frac{\sigma_{ij}}{1-D}\right) - \frac{\nu}{E}\left(\frac{\sigma_{kk}\delta_{ij}}{1-D}\right) \tag{3}$$

where E and ν are the elastic modulus and the Poisson's ratio for the undamaged material, respectively.

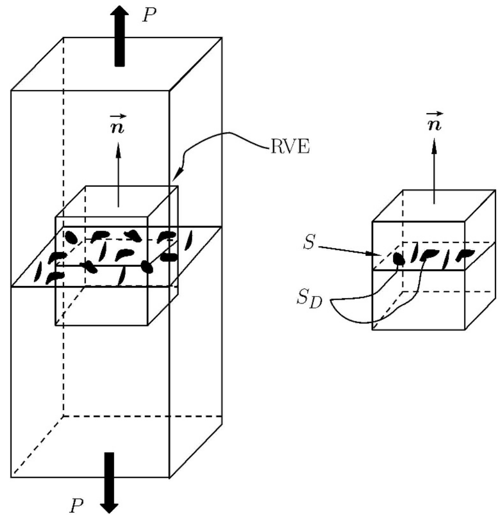


Fig. 1. Representative Volume Element (RVE)[16].

2.2. Damage Evolution Equation

A damage evolution model for high cycle fatigue is proposed by Xiao[21], which has the following form:

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \alpha\left(\frac{\sigma_a}{1-n\sigma_m}\right)^m (1-D)^{-\beta} \tag{4}$$

where σ_a and σ_m represent the stress amplitude and mean stress of the fatigue loading, respectively. α, β, m and n are material parameters determined by using the uniaxial fatigue tests data. The damage evolution model can be applied to performing analysis of high cycle fatigue life in uniaxial cases.

For the purpose of application in multiaxial fatigue problems, the damage evolution model needs to be extended to three-dimensional form. The amplitude and mean value of damage equivalent stress, which are expressed by Eqs.(5), are adopted to describe damage evolution, instead of the stress amplitude σ_a and mean stress σ_m in Eq.(4).

$$\sigma_a^* = \frac{1}{2}\sigma^*\left(\sigma_{ij\,\max} - \sigma_{ij\,\min}\right), \quad \sigma_m^* = \frac{1}{2}\sigma^*\left(\sigma_{ij\,\max} + \sigma_{ij\,\min}\right) \tag{5}$$

where $\sigma_{ij\,\max}$ and $\sigma_{ij\,\min}$ are the maximum and minimum values of stress tensor ij components during a loading cycle, respectively. $\sigma^*(\sigma_{ij})$ is the damage equivalent stress for a stress state and is a function of stress components[10].

$$\sigma^*(\sigma_{ij}) = \sqrt{(\sigma_{11}^2+\sigma_{22}^2+\sigma_{33}^2) - 2\nu(\sigma_{11}\sigma_{22}+\sigma_{11}\sigma_{33}+\sigma_{22}\sigma_{33}) + 2(1+\nu)(\sigma_{12}^2+\sigma_{13}^2+\sigma_{23}^2)} \tag{6}$$

The damage equivalent stress amplitude, σ_a^*, is calculated using the function Eq.(6) with the difference of stress components as its arguments. Namely, the difference between the two stresses, $\sigma_{ij\,\max} - \sigma_{ij\,\min}$, is considered a new stress, the damage equivalent stress of which is calculated to obtain the variable σ_a^*. A similar definition is one for the mean value of damage equivalent stress σ_m^*. The amplitude and mean value of damage equivalent stress can degenerate to the stress amplitude σ_a and mean stress σ_m for uniaxial loading. Then the damage evolution law is expressed in a form similar to Eq.(4)

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \alpha\left(\frac{\sigma_a^\star}{1-n\sigma_m^\star}\right)^m (1-D)^{-\beta} \tag{7}$$

To predict the fatigue life of notched specimens and the Pitch-Change-Link, two things should be done besides yielding the damage evolution equation. The first one is to obtain the material parameters in the damage evolution equation, and the second is to implement the coupling calculation between damage evolution of material and stress distribution in the structure.

III. EXPERIMENTS AND MATERIAL PARAMETERS

Since α, β, m and n in the damage evolution equation are material parameters, they can be obtained according to the fatigue experimental data of smooth specimens. For the uniaxial fatigue with a constant loading amplitude, the number of cycles to failure N_R is obtained by integrating Eq.(4) from $D = 0$ (initial undamaged state) to $D = 1$ (macrocrack initiation)

$$N_R = \frac{1}{\alpha(1+\beta)} \left(\frac{\sigma_a}{1 - n\sigma_m} \right)^{-m} \tag{8}$$

The parameters in the damage evolution equation are determined by using fatigue tests of standard specimens.

3.1. Experiments and Material Parameters for LC4

The middle and high cycle fatigue experimental data of standard LC4 specimens used in this study emanate from[22]. The LC4 aluminum alloy, which is similar to the 7010 alloy, was applied for the fatigue experiments of smooth and U-notched specimens (as shown in Figs.2 and 3, respectively). The plate thickness is 2.5 mm for all specimens and the elastic stress concentration factor K_t of the notched specimens is about 2.0. The chemical composition and the mechanical properties are presented in Tables 1 and 2. The fatigue tests were carried out at four different values of mean nominal stress σ_m ($\sigma_m = 0$, 70, 140 and 210 MPa) and the cycle frequency was $110 \sim 130$ Hz. All the experiments were conducted on an Amsler 1478 fatigue testing machine.

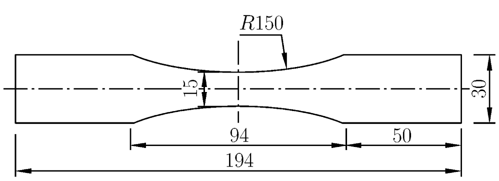


Fig. 2. Smooth LC4 specimen geometry (dimensions in mm).

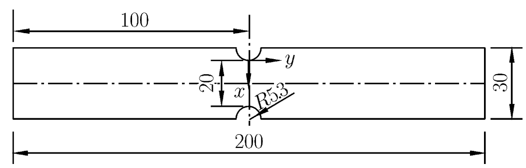


Fig. 3. U-notched LC4 specimen geometry (dimensions in mm).

Table 1. **Chemical composition of LC4 alloy (in weight percent)**

Cu	Mg	Zn	Si	Fe	Cr	Mn	Al
1.60	2.12	6.70	0.14	0.34	0.13	0.28	88.69

Table 2. **Mechanical properties of LC4 plate**

LC4 property	Aged
0.2% Yield stress (MPa)	494
Ultimate stress (MPa)	549
Young's modulus (GPa)	73
The Poisson's ratio	0.33

Four material parameters in the damage evolution equation need to be determined by the fatigue experimental data of standard specimens. Parameters m and $1/[\alpha(1+\beta)]$ in Eq.(8) can be obtained from the fatigue tests data at zero mean nominal stress value ($\sigma_m = 0$). Parameter n is determined from the data at other different mean nominal stresses ($\sigma_m = 70$, 140, 210 MPa). In the present study, the independent parameters α and β will be determined numerically by the method presented in §4.1. In that section, the mean life 50040 of the fatigue test for the notched specimens under the loading condition of $\sigma_m = 0$ and $\sigma_{\max} = 130$ MPa ($\sigma_{\max}$ is the maximum nominal stress applied on the specimen) are chosen to identify the value each of α and β. Finally, the material parameters for LC4 alloy used in this research are listed in Table 3.

Figure 4 shows the comparisons of the experimental data and the predicted S-N curve for smooth plate specimens for $\sigma_m = 0$, 70, 140, 210 MPa. These parameters will be used directly in the fatigue life prediction of notched specimens and the results are presented in § 5.2.

Table 3. **Material parameters of the fatigue damage method for LC4**

α	β	m	n
1.791×10^{-16}	4.0	4.52642	0.00158855

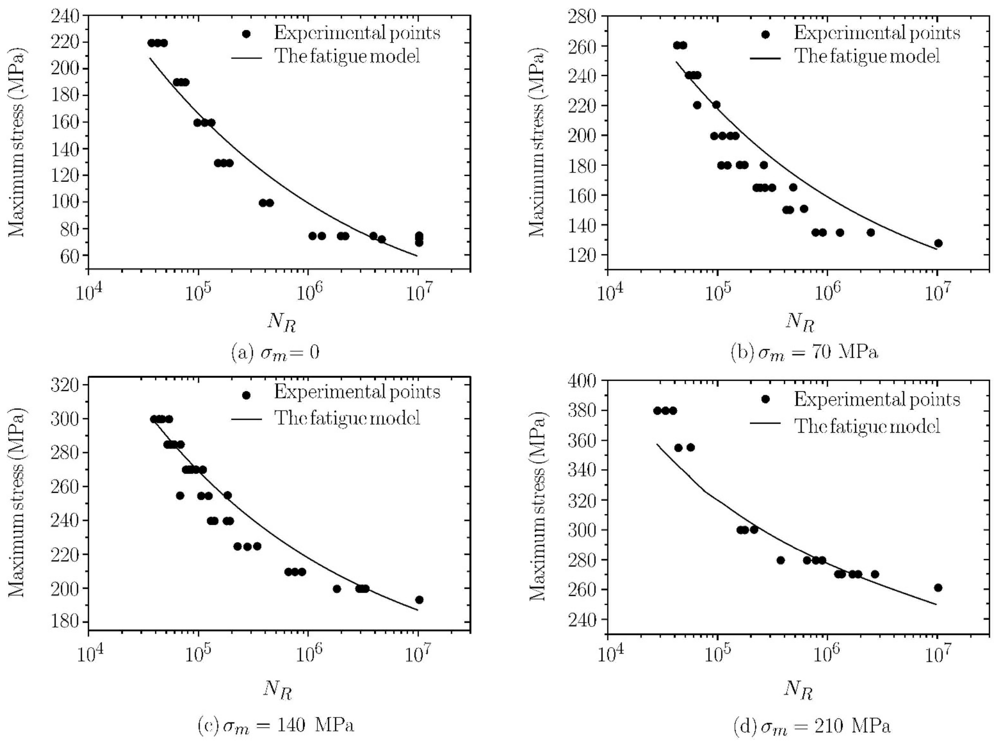


Fig. 4. Experimental points and predicted S-N curve of smooth specimens.

3.2. Experiments and Material Parameters for LY12CZ

The Pitch-Change-Link is an important and familiar component in the rotating system of helicopters. The one in this research is an actual engineering component of an in-service helicopter composed of upper lugs, under lugs and the connecting rod (as shown in Fig.5). The upper lugs and lower lugs are made of 40CrNiMoA, while the connecting rod is made of LY12CZ, which is similar to alloy 2024 . The chemical compositions and the mechanical properties of LY12CZ are presented in Tables 4 and 5. The maximum value of the fatigue experiment load applied on the upper and lower lugs is 30400 N and the minimum value is 2942 N. The detailed test results are published in Ref.[14].

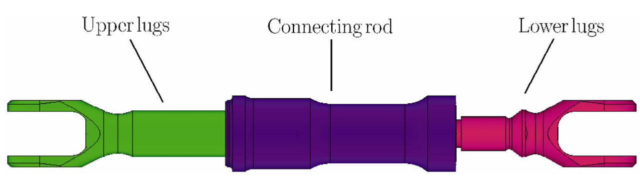


Fig. 5. The geometry model of Pitch-Change-Link.

Table 4. **Chemical compositions of LY12CZ alloy (in weight percent)**

Cu	Mg	Zn	Si	Fe	Mn	Al
4.61	1.54	0.1	0.26	0.29	0.58	92.62

Table 5. **Mechanical properties of LY12CZ plate**

LC4 property	Aged
0.2% Yield stress (MPa)	343
Ultimate stress (MPa)	466
Young's modulus (GPa)	73
The Poisson's ratio	0.3

Table 6. **Material parameters of the fatigue damage method for LY12CZ**

α	β	m	n
3.6583×10^{-15}	4.0	4	7.4487×10^{-4}

Since the material of the connecting rod of Pitch-Change-Link is much weaker than that of the lugs, it is more important to predict the fatigue life of the connecting rod. With reference to the fatigue test data of smooth specimens in Ref.[22], the parameters for LY12CZ are shown in Table 6. During the process, the mean experimental fatigue life data of the notched specimens with $K_t = 2$, $\sigma_m = 70$ MPa, $\sigma_{\max} = 155.6$ MPa is chosen to identify the parameters α and β.

IV. COMPUTATIONAL METHODOLOGY AND FE MODELS

4.1. Damage Mechanics-Finite Element Method

The general purpose FE code ANSYS is used to compute the stress distribution in the notched specimen and the Pitch-Change-Link. The coupling relationship between the damage field and the stress field is taken into account in FE computation. In this research, the APDL language on the ANSYS platform is employed to achieve the implementation and the methodology is illustrated in the flowchart of Fig.6.

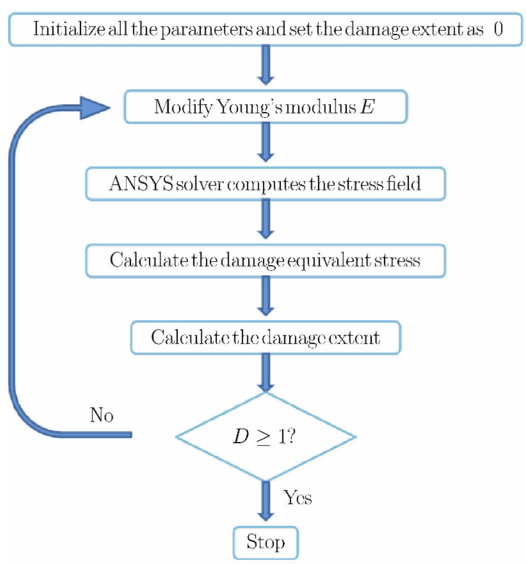


Fig. 6. The flowchart of Damage mechanics-finite element method.

The initial value of the damage D is set to be zero. The subsequent damage is accumulated to calculate the reduction in Young's modulus using the following formula:

$$E^{(i+1)} = E\left[1 - D^{(i+1)}\right] \tag{9}$$

where E stands for Young's modulus of material in the undamaged state. After modifying Young's modulus of each element, ANSYS solver computes the stress field. After that, the damage equivalent stress can be calculated. For the sake of simplicity, the forward difference integration is adopted to calculate the damage increment after ΔN cycles

$$\Delta D^{(i+1)} = \Delta N \cdot \dot{D}^{(i)} \tag{10}$$

$$D^{(i+1)} = D^{(i)} + \Delta D^{(i+1)} \tag{11}$$

Repeat the FE analysis and the accumulation of damage until the maximum value of damage reaches 1, and the number of the cycles is the fatigue crack initiation life.

It is worth mention that the calculation illustrated in Fig.6 is conducted in each element, which has unique damage value. The Young's modulus of each element is computed according to its damage value. The most dangerous element is the one with the maximum value of damage.

4.2. FE Modeling for Notched Specimens

The notched specimen is modeled for the validation of the approach. Only 1/8 of the specimen is built with the symmetric boundary conditions at the plane of symmetry in the ANSYS platform. The bottom surface of model is one of the three planes of symmetry and the cyclic loading is applied on the right side of model (as shown in Fig.7(a)). A higher order 3-D 20-node solid element exhibiting quadratic displacement behavior is employed. It is defined by 20 nodes with three degrees of freedom per node: translations in the nodal x, y, and z-directions. A total of 7500 elements are created and the mesh size of the element at the notch root is about 0.2 mm by 0.2 mm by 0.625 mm, as illustrated in Fig.7(b). Figure 7(c) shows the distribution of axial stress on the notched specimen without damage under a nominal axial stress of 170 MPa. The value of maximum axial stress in the loading direction at the notch tip is 343.549 MPa. The appropriate mesh density is determined by the condition under which a convergent stress solution can be obtained. If the number of elements shown in Fig.7(c) increases twice, the maximum axial stress under the same load will decrease by less than 0.1%. So the mesh density of the model in Fig.7(c) is appropriate. Then the elastic stress concentration factor K_t can be calculated as

$$K_= \frac{343.549}{170} = 2.0207 \tag{12}$$

which is very close to the given stress concentration factor, $K_t = 2$.

4.3. FE Modeling for Pitch-Change-Link

Only the connecting rod of the Pitch-Change-Link is constructed in the ANSYS platform. A 3-D solid element is adopted to simulate the structure and the element is defined by eight nodes each with

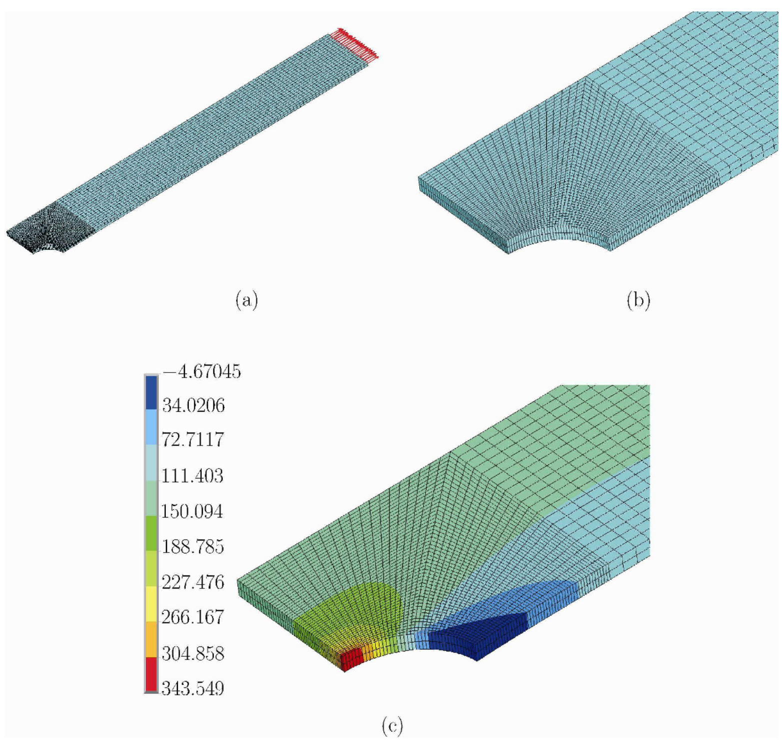


Fig. 7. (a) FE mesh of notched specimen, (b) detailed FE mesh, (c) initial distribution of axial stress under the nominal axial stress of 170 MPa.

three degrees of freedom : translations in the nodal directions, x, y, and z. In total, 63920 elements are generated and the FE model is shown in Fig.8(a). As the right section of model is constrained, the cyclic load is applied on the left section . Figure 8(b) illustrates the axial stress field of the Pitch-Change-Link without damage under the load of 30400 N.

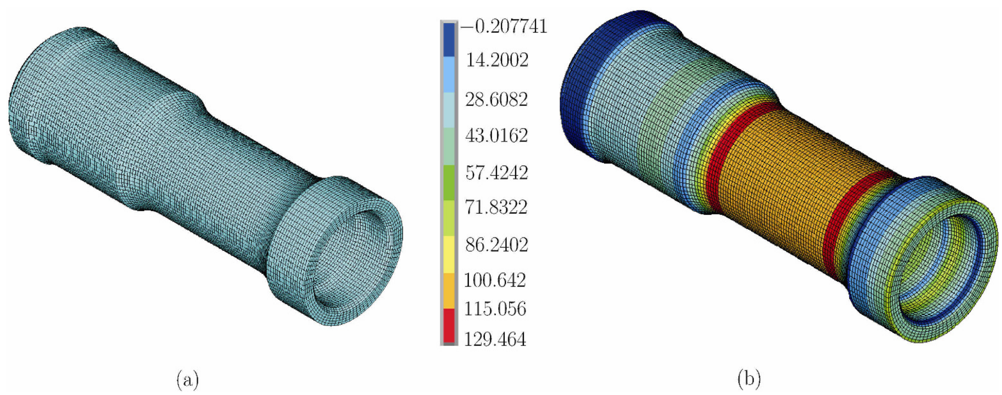


Fig. 8. (a) FE mesh of Pitch-Change-Link, (b) initial distribution of axial stress under the load of 30400 N.

V. RESULTS AND DISCUSSION

5.1. Damage Accumulation Property

The loading and damage extent in the damage evolution equation are separable variables when the stress distribution is uniform along the cross section of structure. So the damage evolution equation can be integrated into a closed form. According to Eq.(8), the relation between N/N_R and D can be derived as

$$\frac{N}{N_R} = 1 - (1 - D)^{\beta+1} \tag{13}$$

Under the above condition, for multi-level fatigue loadings, the damage evolution equation will lead to linear damage accumulation, which is the shortcoming of this model. For most cases, the stress distribution is not uniform along the cross section of structure. Then stress distribution varies as the number of cycle increases and the damage evolution equation cannot be integrated direct. Therefore, at this time the damage model is a nonlinear damage accumulation model. To illustrate this property, three experiments on notched specimens are calculated with the adopting of damage mechanic-finite element method. The loading conditions are $\sigma_m = 0$, $\sigma_{\max} = 130$ MPa; $\sigma_m = 70$ MPa, $\sigma_{\max} = 180$ MPa and $\sigma_m = 140$ MPa, $\sigma_{\max} = 200$ MPa, respectively. Figure 9 shows the damage evolution of the most dangerous element, which is at the notch tip for the three tests. The three curves in Fig.9 are not identical, which shows the effect of loading sequence on fatigue life is considered in this approach.

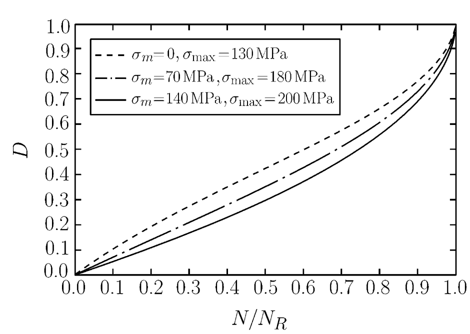


Fig. 9. Damage evolution of most dangerous element for three tests.

5.2. Results of Notched Specimens

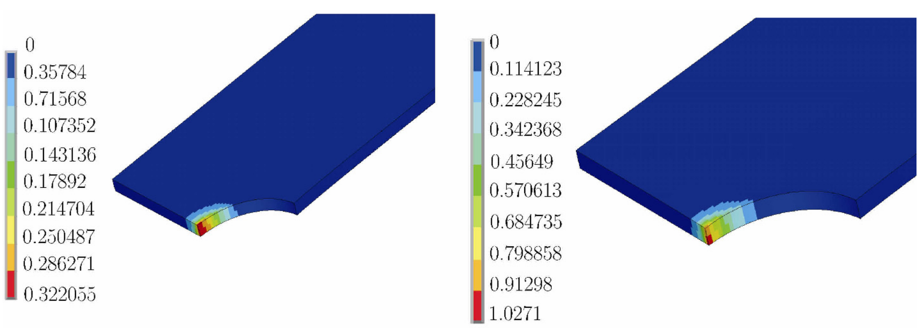
Two methods can be used to compute the fatigue life of the notched specimens. The first one is the integrated approach, in which the stress amplitude and mean stress of the tests are are assumed to be constant, and Eq.(7) is integrated into a closed form

$$N_R = \frac{1}{\alpha(1+\beta)}\left(\frac{\sigma_a^*}{1-n\sigma_m^*}\right)^{-m} \tag{14}$$

However, Eq.(19) will predict conservative lives since the method does not take the coupling relation between the damage field and the stress field into account. The second one is the damage mechanics-finite element method which considers the coupling relation and is adopted here. Additionally, it is necessary to do convergence validation for fatigue life with the damage mechanics-finite element method.

For the notched specimens, due to the stress concentration and the Poisson's ratio effect at the notch tip, the stress along the thickness direction is greater at the mid-thickness position than that at the two edges of the notch tip. The damage is severer at that position. This is illustrated in Figs.10(a) and (b), which show the damage field of specimen under the loading condition of $\sigma_{\max} = 170$ MPa and $\sigma_m = 0$ when $N = 0.53N_R$ and when $N = N_R$, respectively. The element with the maximum damage extent value is near the mid-thickness position and localized at the notch tip. As the number of cycle increases, the equivalent Young's modulus of element at the notch tip will decrease, as shown in Fig.11, resulting in the redistribution of the stress. When N reaches N_R, the damage value of the most dangerous element is close to 1.

Figure 12 compares the results of the experimental data and the predicted fatigue lives. The predicted fatigue lives are in good agreement with the fatigue tests data of the middle or high cycle. For the low cycle fatigue lives, plastic deformation will occur in the specimens which has not been considered in our model presently. Hence deviations of predicted lives from experimental data. The maximum error



(a) $N = 0.53N_R$ (b) $N{=}N_R$ ($\sigma_{\max}$=170 MPa, σ_m= 0)

Fig. 10. Damage field.

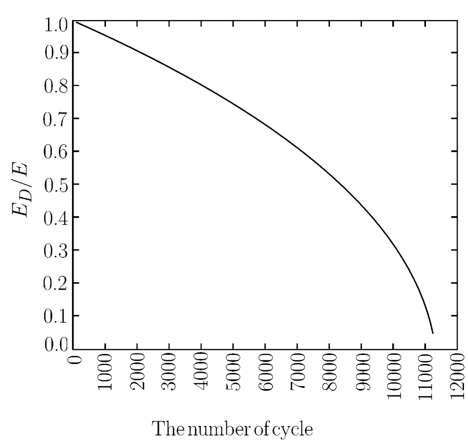
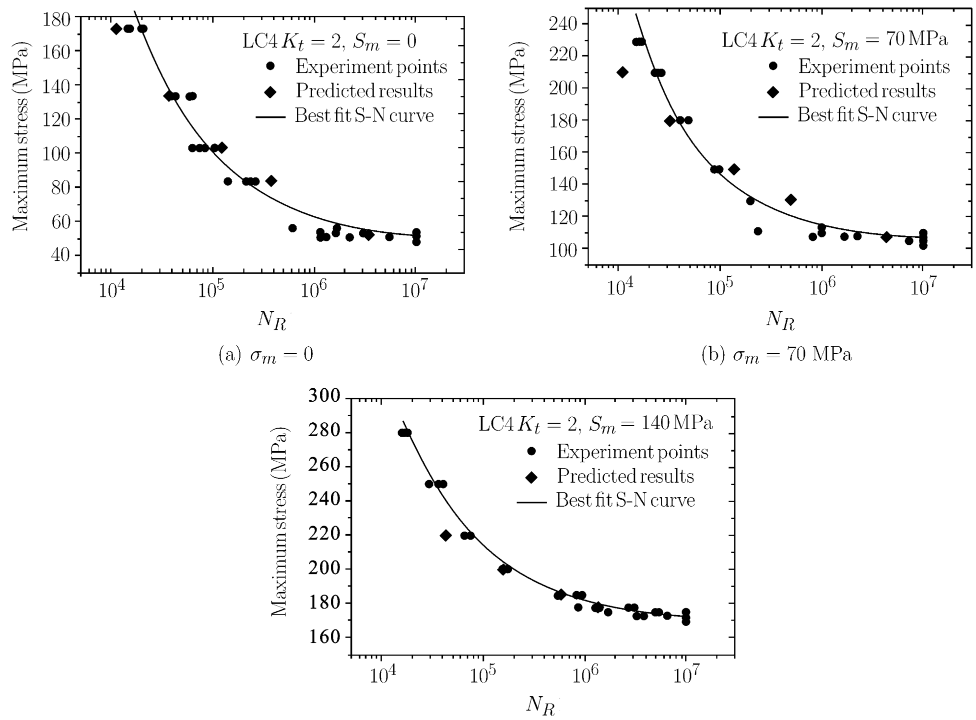


Fig. 11. Reduction in the equivalent Young's modulus of element at the notch tip ($\sigma_{\max} = 170$ MPa, $\sigma_m = 0$).



(a) $\sigma_m = 0$

(b) $\sigma_m = 70$ MPa

(c) $\sigma_m = 140$ MPa

Fig. 12. Experimental and predicted lives for the notched specimens.

between logarithm fatigue life of the predicted results and the experimental mean lives for the middle and high cycle fatigue is 7.95%.

5.3. Results of Pitch-Change-Link

The tension-tension fatigue experiments for Pitch-Change-Link have been conducted[14] when the maximum load is 30400 N and the minimum value is 2942 N. Three experimental fatigue lives of the connecting rod are valid whose values are 9.12×10^6, 2.16×10^5, 5.47×10^6, respectively. The Pitch-Change-Link is an actual in-service structure that is more complicated than the standard specimen. As the number of fatigue test samples is limited, the scatter of experimental fatigue lives is observable.

The result of the prediction life is obtained with reference to the parameters shown in Table 6. The comparison between the experimental mean life and the calculated mean life is listed in Table 7. The calculated fatigue life is about 1.5 times longer than the mean experimental fatigue life. The predicted result is acceptable for engineering application.

Table 7. **The experimental result and the calculated result of Pitch-Change-Link**

	Component	Fatigue life
Test result	Connecting rod	9.12×10^6, 2.16×10^5, 5.47×10^6
Calculated result	Connecting rod	7.05×10^6

VI. CONCLUSIONS

In this study, a fatigue damage model is created to predict the fatigue life of structures under multiaxial fatigue loading on the basis of damage mechanics. According to thermodynamics, a new damage evolution equation is proposed which considers the effects of both the stress amplitude and mean stress. The parameters in the damage evolution equation determined by the fatigue test data of standard specimens are directly adopted in the fatigue life prediction for notched specimens and actual engineering structures. By integrating the fatigue model into the FEM through further development of the APDL language in ANSYS, a damage mechanics-finite element method is proposed to predict the fatigue life of notched specimens and engineering structures. The calculated fatigue lives of the standard notched specimens are in good agreement with the experimental data. For the in-service structure Pitch-Change-Link, the predicted value of fatigue crack initiation life is acceptable for engineering application. Some other main findings are:

(1) The damage mechanical-finite element method considers the coupling relation between damage and stress distribution, which greatly smoothes the local stress concentration at the near surface location.

(2) For the case of non-uniform stress distribution, this model is a non-linear damage accumulation model. It takes the effect of loading sequence on fatigue life into account by the damage mechanics-finite element method. But the efficiency of the proposed approach for the case of variable amplitude loadings should be examined, which is beyond the scope of this research.

(3) This work mainly investigates fatigue problems of middle and high cycles. Further work will address the FE-based damage mechanics with plasticity.

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